WAVE PROPAGATION ANALYSIS FOR FLEXIBLE PAVEMENTS AND ITS APPLICATION TO BACKCALCULATION ANALYSIS

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ACKNOWLEDGMENT

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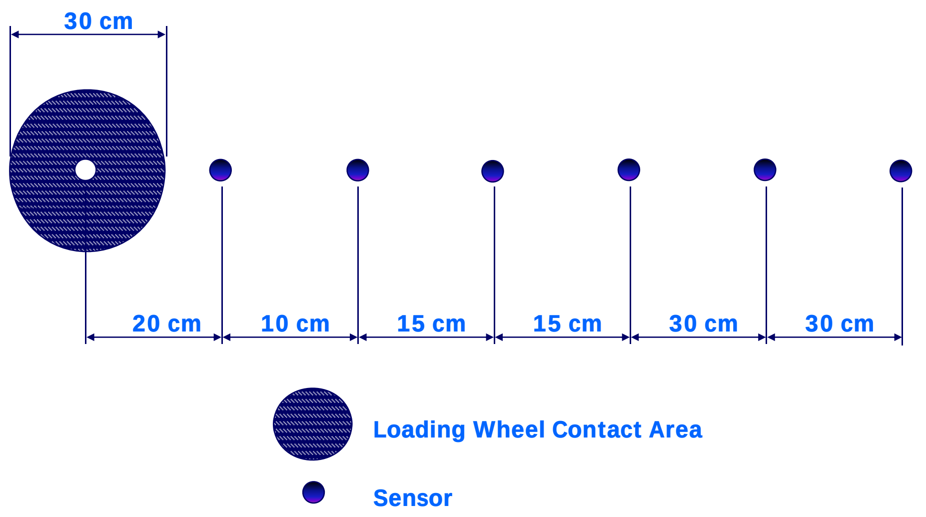
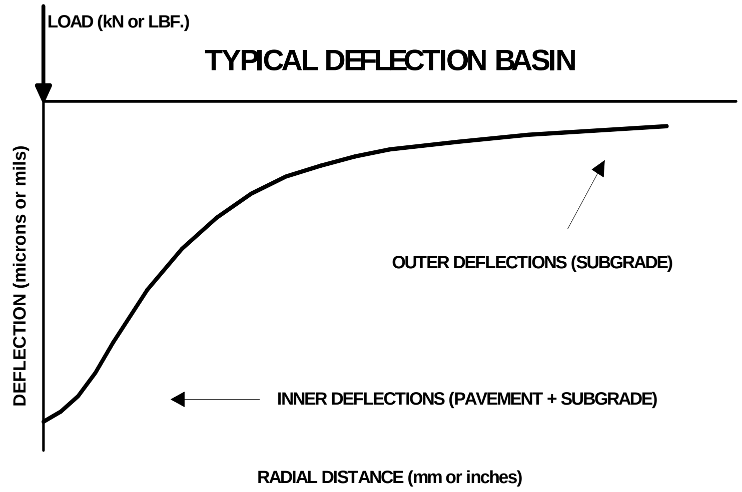
Sample FWD set-up





Axi-symmetric wave propagation analysis

Equation of wave propagation

$$\left.\begin{aligned}\frac{\partial \sigma_r}{\partial r}+\frac{\partial \tau_{rz}}{\partial z}+\frac{\sigma_r-\sigma_\theta}{r}&=\rho\frac{\partial^2 u}{\partial t^2}\\ \frac{\partial \tau_{rz}}{\partial r}+\frac{\partial \sigma_z}{\partial z}+\frac{\tau_{rz}}{r}&=\rho\frac{\partial^2 w}{\partial t^2}\end{aligned}\right\}\quad \text{(Eq. 1)}$$

ρ Layer Density

Strain-Displacement

$$\left.\varepsilon_r=\frac{\partial u}{\partial r},\varepsilon_\theta=\frac{u}{r},\varepsilon_z=\frac{\partial w}{\partial z},\gamma_{rz}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right\}\quad \text{(Eq. 2)}$$

Stress-Strain for Voigt Model

$$\begin{Bmatrix}\sigma_r\\ \sigma_\theta\\ \sigma_z\\ \tau_{rz}\end{Bmatrix}=E\begin{bmatrix}a+2b & a & a & 0\\ a & a+2b & a & 0\\ a & a & a+2b & 0\\ 0 & 0 & 0 & b\end{bmatrix}\begin{Bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \varepsilon_z\\ \gamma_{rz}\end{Bmatrix}+F\begin{bmatrix}a+2b & a & a & 0\\ a & a+2b & a & 0\\ a & a & a+2b & 0\\ 0 & 0 & 0 & b\end{bmatrix}\frac{d}{dt}\begin{Bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \varepsilon_z\\ \gamma_{rz}\end{Bmatrix}\quad \text{(Eq. 3)}$$

$$a=\frac{\nu}{(1+\nu)(1-2\nu)}\qquad b=\frac{1}{2(1+\nu)}\qquad F=\beta E$$

Stress-Strain for Complex Modulus

$$\begin{Bmatrix}\sigma_r\\ \sigma_\theta\\ \sigma_z\\ \tau_{rz}\end{Bmatrix} = \left(E + i\tilde{E}\right)\begin{bmatrix} a+2b & a & a & 0\\ a & a+2b & a & 0\\ a & a & a+2b & 0\\ 0 & 0 & 0 & b\end{bmatrix}\begin{Bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \varepsilon_z\\ \gamma_{rz}\end{Bmatrix} \qquad \text{(Eq. 4)}$$

$$a = \frac{\nu}{(1+\nu)(1-2\nu)} \qquad b = \frac{1}{2(1+\nu)}$$

DERIVATION PROCESS

Step 1: apply Fourier Transform

Step 2: use Hankel Transform

Step 3: Solve for two displacement components

Step 4: use Inverse Hankel Transform

Step 5: apply Inverse Fourier Transform

PAVEMENT PROFILE

$$P(t) = 49\sin^2\left(\frac{\pi}{0.04}\right)t \text{ kN}$$

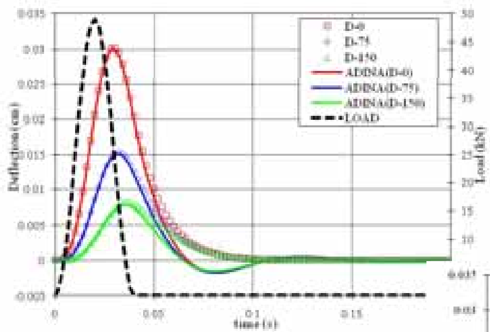
$$0 \le t \le 0.04\text{s}$$

15 cm E1=5000 MPa ρ=2200 kg/m^3, F_1=50MPa·s, υ_1=0.35

35 cm E2=400 MPa P=2000 kg/m^3 , F_2=4MPa·s, υ_2=0.35

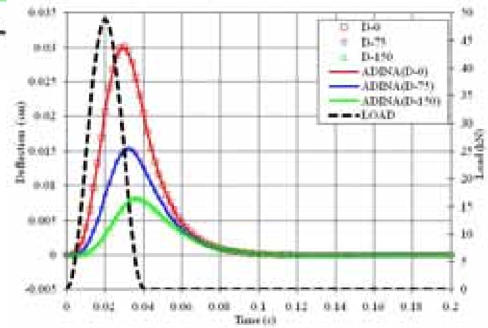
E3=200 MPa ρ=1600 kg/m^3, F_3=0.6MPa·s, υ_3=0.35

COMPARISON OF WAVE-PALS AND ADINA

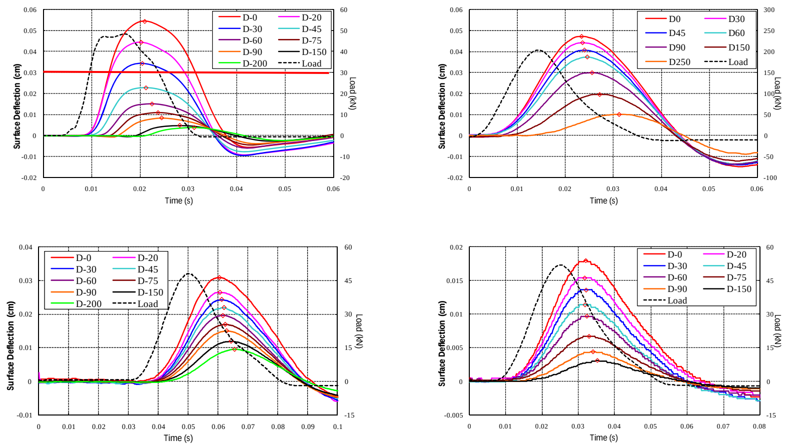


Analysis Domain (R=5m, Z=6m)

Analysis Domain (R=15m, Z=15m)



SAMPLES OF FWD DATA



EVALUATION FUNCTION

$$J = \frac{1}{2}\sum_{\lambda=1}^{L}\sum_{i=1}^{I}\sum_{k=1}^{K}\left\{u_i^{(\lambda)}(t_k) - z_i^{(\lambda)}(\mathbf{X}, t_k)\right\}^2$$

L : no. of data sets
I : no. of sensors
K : no. of discrete time steps
u : measured deflections
z : computed deflections using measured load
X : unknown parameters

BACKCALCULATION

TABLE 1 Details of pavement sections at FAA facility

Section 1 (LFS)

	Material	Density(kg/m^3)	Poisson's ratio	Thickness (m)	Range of seed modulus (MPa)
Layer 1	Asphalt	2300	0.35	0.251	5,000 – 10,000
Layer 2	Base	2242	0.35	0.752	100 – 500
Layer 3	Sub-grade	1949	0.4		20 - 100

- VOIGT MODEL(V-V-V)
- COMPLEX MODULUS(C-C-C)

BACKCALCULATION

Section 2 (LFC)

	Material	Density(kg/m^3)	Poisson's ratio	Thickness (m)	Range of seed modulus (MPa)
Layer 1	Asphalt	2300	0.35	0.127	5,000 – 10,000
Layer 2	Base	2545	0.35	0.197	100 – 500
Layer 3	Subbase	2242	0.35	0.924	100 - 500
Layer 4	Subgrade	1949	0.4		20 - 100

- VOIGT MODEL(V-V-V-V)
- COMPLEX MODULUS(C-C-C-C)

BACKCALCULATION

Section 3 (HFC)

	Material	Density(kg/m^3)	Poisson's ratio	Thickness (m)	Range of seed modulus (MPa)
Layer 1	Asphalt	2300	0.35	0.133	5,000 – 10,000
Layer 2	Base	2545	0.35	0.276	100 – 500
Layer 3	Sub-grade	2095	0.4		100 - 500

- VOIGT MODEL(V-V-V)
- COMPLEX MODULUS(C-C-C)

STATIC BACKCALCULATION

TABLE 2 Static backcalculation

Section	Pavement surface temperature (°C)		Modulus (MPa)			
			Layer 1	Layer 2	Layer 3	Layer 4
1	16	Results	10345	-	148	186
		Std deviation	0.32	-	0.03	0.03
2	15	Results	10289	547	134	174
		Std deviation	0.16	0.02	0.02	0.04
3	15	Results	13713	153	396	-
		Std deviation	3.73	0.05	0.08	-

DYNAMIC BACKCALCULATION

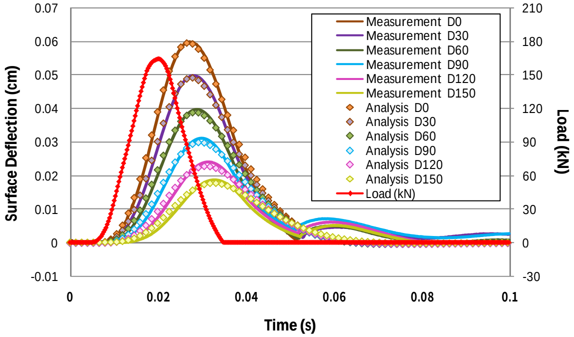
TABLE 3 Dynamic backcalculation (Voigt model)

Section	Pavement surface temperature (°C)		Modulus (MPa)				Damping (MPa·s)			
			Layer 1	Layer 2	Layer 3	Layer 4	Layer 1	Layer 2	Layer 3	Layer 4
1	16	Results	6631	-	136	102	47	-	1.0	0.3
		Std deviation	461	-	25	9.5	7.0	-	0.27	0.08
2	15	Results	10113	141	137	102	67	0.9	0.8	0.2
		Std deviation	288	15	4.7	4.3	2.0	0.07	0.05	0.06
3	15	Results	10416	138	353	-	63	0.7	0.7	-
		Std deviation	110	4.2	4.7	-	1.9	0.06	0.05	-

DYNAMIC BACKCALCULATION

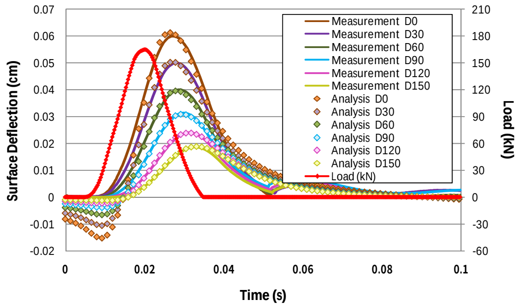
TABLE 4 Dynamic backcalculation (complex modulus)

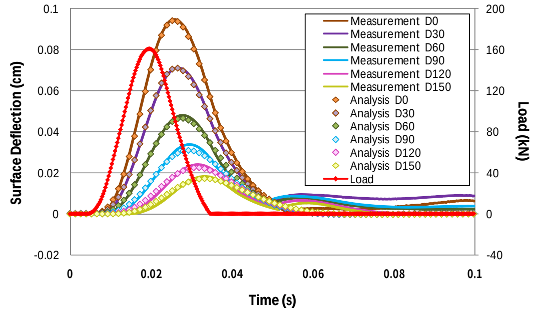
Section	Pavement surface temperature (°C)		Real part (MPa)				Imaginary part (MPa)			
			Layer 1	Layer 2	Layer 3	Layer 4	Layer 1	Layer 2	Layer 3	Layer 4
1	16	Results	3947	-	172	83	601	-	95	24
		Std deviation	279	-	11	4.5	346	-	19	3.3
2	15	Results	5932	156	144	73	7536	148	76	18
		Std deviation	374	34	3.8	2.6	310	22	7.5	1.1
3	15	Results	7581	161	308	-	7684	92	107	-
		Std deviation	175	6.2	4.7	-	272	9.9	4.4	-



Section-1: Voigt Model

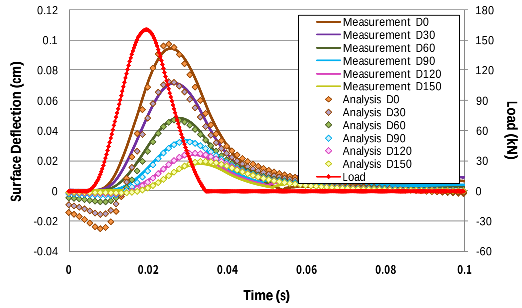
Section-1: Complex Modulus Model

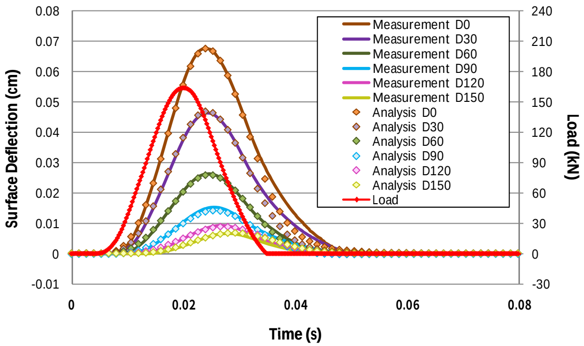




Section-2: Voigt Model

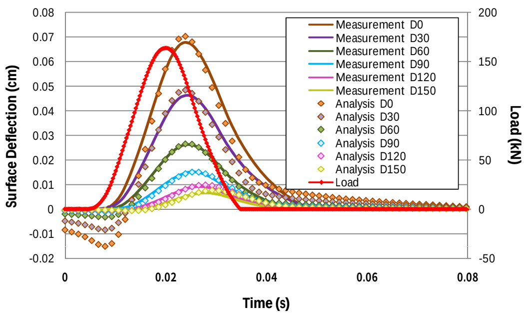
Section-2: Complex Modulus Model





Section-3: Voigt Model

Section-3: Complex Modulus Model



Conclusions

- **Small values of CoV from the backcalculation analysis using 50 sets of seed values confirms the reliability of Wave BALM software.**
- **Based on the comparison of measured and computed deflections, Voigt model is more suitable for application on FWD data analysis than complex modulus model.**
- **Damping component for dynamic pavement analysis subjected to impact load must be frequency dependent. Frequency independent model will be suitable only when the load is harmonic.**